MAT JOURNALS

Journal of Signal Processing
Volume 5 Issue 2

Introduction to Laplace Transform with its Application in Engineering Signals and Electrical Circuit

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DOI: *http://doi.org/10.5281/zenodo.2658890*

Abstract

An introduction to Laplace transform and its application in engineering field is the topic of this study. It deals with what is the Laplace transform, and what are the applications of Laplace transform. The definition of Laplace transform is given and most of the applications are mentioned. A few practical life applications are also given in the manuscript. Laplace transform is a powerful mathematical tool used by the engineers and scientists. It is useful to solve linear differential equations with given initial conditions by using algebraic methods, to solve the electrical circuits with given initial conditions, useful in quantum physics. The concept of Laplace transform are applied in area of science and technology.

Keywords:*Frequency domain, laplace transform, linear differential equations, non-periodic, time domain, etc.*

INTRODUCTION

Analysis of engineering field and linear differential equation is performed by using Laplace transformation. Laplace transform provide algebraic method to analyse linear system.The process of Laplace transform is:1)first convert system transfer function. 2)Convert differential equation is s-domain equation.3)Finding the output function which is combination of input function and transfer function. 4)Using partial functions to reduce the output function to simpler components.5)Conversion of output equation back to time domain. As per the Dirichlet conditions,for any function of time t i.e.,

f(t) to be Laplace transformable. It must satisfy the following mentionDirichlet conditions.

- f(t) is the piecewise continuous which is the single valued but f(t) have a finite number of finite isolated discontinuities for positive value of t.
- f(t) is the exponential order that is f(t) is less than Se $^{-a_0 t}$ as t tends to ∞ where, s >0 which is constant and a_o>0 which is a real number.

For any function f(t), it must satisfies the Dirichlet conditions , then

F(s) = $\int_0^{\infty} f(t)\, e^{-st}\, dt$

F(s) is Laplace transformation of f (t). Where, S is a real variable or a complex variable.

For$\int |f(t) e^{-st}|dt <$ ∞, the integral $\int f(t) e^{-st}\, dt$ must be converges

Some Important Properties of Laplace Transforms

Given the functions f (t) and g (t) and their respective Laplace transforms F (s) and G (s),

f (t) = L^{-1} [F (s)], g (t) = L^{-1} [G (s)]

The following is a list of properties of unilateral Laplace transform.

- Linearity
  L [a f (t) + b g (t)] = a F (s) + b G (s)

- Frequency differentiation
  L [t f (t)] = -F (s)
  L [t^n f (t)] = $(-1)^n$ F^n (s)
- Differentiation

L [f'] = s L [f] – f (0^-)
L[f"] = s^2 L [f] – s f (0^-) – f(0^-)
L [$f^{(n)}$] = s^n L [f] – s^{n-1} f (0^-)……………- f^{n-1} (0^-)

Frequency integration

$$L\left[\frac{f(t)}{t}\right] = \int_{s}^{\infty} F(\sigma)\, d\sigma$$

Frequency shifting

L[e^{at} f (t)] = F (s – a)
L^{-1} [F (s – a)] = e^{at} f (t)

Time shifting

$$L[f(t-a)u(t-a)] = e^{-as}F(s)$$
$$L^{-1}[e^{-as}F(s)] = f(t-a)u(t-a)$$

Where, u (t) is the Heaviside step function.

Fundamental Relationships

Laplace transformation is considered as mainly two sided spectrum, which is combination of one sided spectrum.Conversion of Laplace transformation into any other transform is much easier.
Example: Laplace transform converted into Fourier transform is as follow

Relationship between Laplace transform and Fourier transforms

The continuous Fourier transform is equivalent to evaluating the bilateral Laplace transform with complex argument s = iw

$$F(\boldsymbol{\omega}) = F[f(t)] = L[f(t)]/_{s=iw}$$
$$= F(s)/_{s=iw}$$
$$= \int_{-\infty}^{\infty} e^{-iwt} f(t)dt$$

Applications of Laplace Transform

The applications of Laplace transform in the domain of engineering are mention given below. Laplace transform has so many applications in almost all engineering disciplines such as system modelling, analysis of electrical and electronic circuits, digital signal processing and process controls, automation and many more. In Laplace transform, critical application is process controls. It helps to analyze the variables, which when altered, produces desired manipulations in the result. For example, while studying theelectrical circuits and machines, we have to consider the resisters, inductor and capacitors. We have to calculate current flowing through all the components.

Electric Circuit Theory Example 1

The switching transient phenomenon in the RL, RC or RLC circuits can be solved by Laplace transform. Let us consider a series RLC circuit as shown Fig. 1 to which a DC voltage V_0 is suddenly applied.

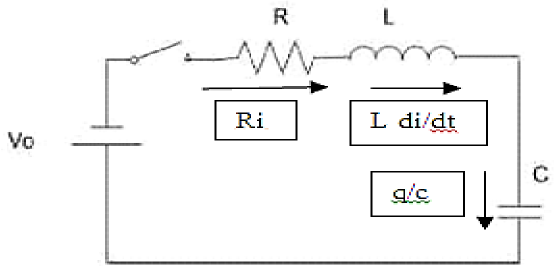

Figure 1: *Series RLC circuit.*

Now applying Kirchoff's voltage law (KVL) to the circuit, we have

$$R_i + L\frac{di}{dt} + \frac{1}{C}\int i\, dt = V_0$$

Differentiating bothsides,

$$L\frac{d^2 i}{dt^2} + \frac{1}{Ci} + R\frac{di}{dt} = 0 \ or$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \rightarrow (1)$$

Applying Laplace transform to this equation, let us assume that the solution of this equation is
i (t) = K e^{sT}

Where, K and s are constants which may be real, imaginary or complex. From equation (1)

$$L K s^2 e^{sT} + R K e^{sT} + \frac{1}{C K} e^{sT} = 0$$

Which on simplification gives

$$s^2 + \frac{R}{L} s + \frac{1}{L C} = 0$$

The two roots of this equation would be

$$S_1, S_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

The general solution of the differential equation is thus,

$$i(t) = K_1 e^{s_1 T} + K_2 e^{s_2 T}$$

Where, K_1 and K_2 are determined from the initial conditions.

Now, if we define α = Damping coefficient = R / 2 L and

Natural frequency $w_n = \frac{1}{\sqrt{LC}}$

Which is also known as undamped natural frequency or resonant frequency.

Thus, the roots are

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_n^{\,2}}$$

The final form of solution depends on whether

$$\left(\frac{R^2}{4L^2}\right) > \frac{1}{LC} \quad \left(R = \frac{1}{LC}\right)$$

CONCLUSION

This paper consist of a brief overview of what is the Laplace transform and its application. The primary application of Laplace transform is converting a time domain function into its frequency domain function. Relations of Laplace transform with Fourier transforms is mentioned and it gives the application of Laplace transformin science andengineering field.

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Cite this article as: *Chopda P.P., & Rayate S. J. (2019). Introduction to Laplace Transform with its Application in Engineering Signals and Electrical Circuit. Journal of Signal Processing, 5(2), 9–11. http://doi.org/10.5281/zenodo.2658890*

