MAT JOURNALS

Journal of Signal Processing
Volume 3 Issue 2

Review on Analysis of Low Pass Finite Impulse Response Filter Using Window functions

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Abstract

Finite impulse response (FIR) filter plays a pivotal role in digital signal processing, multirate signal processing and speech analysis in the communication field. Implementation of the FIR filter employing MATLAB simulation tool can ease the computational complexity and enhance the filter performance to a greater extent. This review paper is based on the analysis of low pass FIR (Finite Impulse Response) filter using different windowing techniques. Rectangular window, Hamming window and Kaiser windows are basically considered for our simulation work . MATLAB programming tools are used to characterize the magnitude and phase response of low pass FIR filter and then analyze the input and output signal in frequency domain as well as time domain for the three window functions under consideration.

Keywords: *FIR, Rectangular Window, Hamming Window, Kaiser Window,LTI, FFT, IFFT.*

INTRODUCTION

Filter is primarily a frequency selective network used to remove unwanted component of a signal, such as noise. Digital filter is far better than analog filters because of their inherent stability, reliability and precision accuracy considerably. Communication, image processing, speech processing, signal procesing and synthesis are important application areas of digital filters. There are two types of digital filters: FIR (Finite Impulse Response), IIR (Infinite Impulse Response). Compared to IIR filter, the FIR filter is inherently non-recursive, where the finite precision mathematical error is negligibly small. Its counterpart IIR filter is recursive (with feedback). Owing to its feedback mechanism in the operation of IIR filter, unwanted oscillations may occur. FIR filter gives better amplitude and linear phase characteristics and also avoids the drift, noise and distortion as compared to IIR filter counterparts. The finite impulse response (FIR) filter is one of the most basic elements in any digital signal processing system, with a potential to guarantee a strict linear phase frequency characteristic with any kind of amplitude frequency characteristic. FIR is also used for higher order filter design to meet the design specifications.

SPECIFICATIONS OF FIR FILTERS

Transfer function of a linear time invariant(LTI) filter is expressed as[1]:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0+b_1z^{-1}+---+b_Nz^{-N}}{1+a_1z^{-1}+---+a_Mz^{-M}} \qquad \text{------- (1)}$$

Since FIR filter is non-recursive structure it means it has no feedback, so denominator part of it becomes unity. Then FIR filter of order N can be expressed as[2] :

$$Y[n] = b_0x(n) + b_1x(n-1) + \text{---------} + b_{N-1}x(n-N+1)$$

$= \sum_{k=0}^{N-1} b_k x(n-k)$ ----- (2)

Where, b_k = Filter co-efficient.

Transfer function of FIR filter expressed as polynomial of degree N-1 in the variable z^{-1} with impulse response of h(n) expressed as[3]:

$H(z) = \sum_{k=0}^{N-1} h(k)\, z^{-k}$ ----- (3)

Where h(k) = Impulse response
Since all poles represented at the origin, this characteristic of transfer function shows that FIR filter is much more potentially stable.
FIR filter gives linear phase characteristic if [4]:

$h(n) = \pm h(N-1-n)$ ----(4)

It is the necessary condition for linear phase FIR filter with frequency response characteristic [4]:
For symmetric condition, frequency response can be written as :
$H(\omega) = H_r(\omega) e^{-j\omega\left(\frac{N-1}{2}\right)}$ ----(6)

Where, $H_r(\omega)$ = Real function of frequency response and given as :

$$H_r(\omega)=\begin{bmatrix} h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-n}{2}} h(n)\cos\omega\left(\frac{N-1}{2}-n\right) & \text{N odd} \\ 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\cos\omega\left(\frac{N-1}{2}-n\right) & \text{N even} \end{bmatrix} \text{-(7)}$$

Phase :

$$\varphi(\omega) = \begin{bmatrix} -\omega\left(\frac{N-1}{2}\right) & \text{for } H_r(\omega) > 0 \\ -\omega\left(\frac{N-1}{2}\right) + \pi & \text{for } H_r(\omega) < 0 \end{bmatrix} \text{------(8)}$$

For anti-symmetric condition, frequency response:

$H(\omega) = H_r(\omega) e^{j\left(-\omega\left(\frac{N-1}{2}\right)+\frac{\pi}{2}\right)}$ ----(9)

Where, $H_r(\omega)$ = Real function of frequency response and described as :

$$H_r(\omega)=\begin{bmatrix} 2\sum_{n=0}^{\frac{N-n}{2}} h(n)\sin\omega\left(\frac{N-1}{2}-n\right) & \text{N Odd} \\ 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\sin\omega\left(\frac{N-1}{2}-n\right) & \text{N Even} \end{bmatrix} \text{--(10)}$$

Phase :

$$\varphi(\omega) = \begin{bmatrix} \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) & \text{for } H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{N-1}{2}\right) & \text{for} H_r(\omega) < 0 \end{bmatrix} \text{---(11)}$$

From above specification and exhaustive analysis of FIR and IIR filter, it is clear that FIR have more advantages as compared to IIR.

DESIGN OF FIR FILTER:

Design of FIR filter is based on different popular methods that described as[8] :
Fourier series method
Frequency sampling method
Window technique
Fourier series method has a disadvantage of oscillation that produces Gibbs Phenomenon.
GIBB'S PHENOMENON: For a FIR filter, as a result of truncation to obtain linear phase nature, oscillation occurs due to the slow convergence of Fourier series. This is known as 'Gibbs Phenomenon'.
Frequency sampling method can be preferred for non prototype irregular shape of magnitude response. Demerit of such method is that only at sampled point, desired frequency response is equal to the input frequency response. Interpolation may cause errors to creep into the calculations.
Window technique is far more superior to other methods because it is easy to implement, understandable and also it is free from oscillation as well as ringing effect.

PROCEDURE TO DESIGN FIR FILTER USING WINDOW METHOD :

First define the edge frequencies of passband and stopband also provide a sampling frequency.
The order of filter by using ripples and cutoff frequencies of passband and stopband is determined. Designing of a low pass FIR filter involves following steps :
Let, ideal frequency response of low pass FIR filter[5][4]:

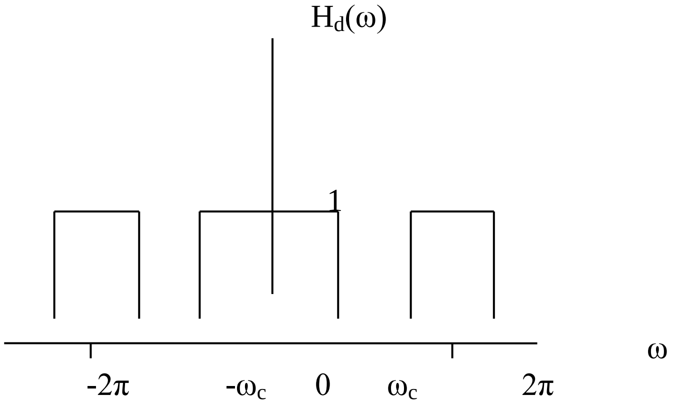

Fig. 1: *Frequency response of ideal low pass filter*

$$H_d(\omega) = \begin{pmatrix} 1 & 0 \le |\omega| \le \omega_c \\ 0 & \omega_c \le |\omega| \le \pi \end{pmatrix} \qquad \text{---(12)}$$

Filter co-efficient of FIR filter can be calculate by the inverse fourier transform of ideal frequency response $H_d(e^{j\omega})$[1]:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n} d\omega \qquad \text{----(13)}$$

$$h_d(n) = \frac{1}{2\pi}\left[\int_{0}^{\omega_c} 1 * e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} 0 * e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1.\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \begin{bmatrix} \frac{2f_c \sin(n\omega_c)}{n\omega_c} & n \neq 0 \\ 2f_c & n = 0 \end{bmatrix} \quad \text{----(14)}$$

Calculate the impulse response co-efficient of FIR filter as[1] :

$$h_d(n) = h(n).w(n) \quad \text{-----(15)}$$

Where, w(n) = window co-efficient (i.e. different for different window technique).
After that draw the magnitude and phase response of FIR filter.

WINDOW TECHNIQUE :
In this paper three window techniques are specifically used to design a low pass FIR filter:
Rectangular window
Hamming window
Kaiser window

Rectangular window:

Weighing co-efficient of rectangular window[2]:

$$W[n] = \begin{bmatrix} 1 & n \leq \frac{N-1}{2} \\ 0 & \text{Otherwise} \end{bmatrix} \quad \text{---(16)}$$

This window technique has not frequently use because of less stopband attenuation and also gives more ripples.

Hamming window:

Hamming window represented by the weighing function[2]:

$$W[n]=\begin{bmatrix} 0.54 - 0.46\cos\frac{2\pi n}{N-1} & 0 \leq n \leq N-1 \\ 0 & \text{Otherwise} \end{bmatrix} \text{-(17)}$$

Hamming window have an advantage over rectangular window of less ripple and large width of first side lobe but it have a demerit of high width of transition region.

Kaiser window
This is described by the weighing function[4]:

$$W[n]=\begin{bmatrix} \frac{I_0(\beta)\left[\sqrt{\left(\frac{N-1}{2}\right)^2-\left(n-\frac{N-1}{2}\right)^2}\right]}{I_0\left[\beta\left(\frac{N-1}{2}\right)\right]} & 0 < n \leq \frac{N-1}{2} \\ 0 & \text{Otherwise} \end{bmatrix} \text{--(18)}$$

where $I_0(\beta)$ = Bessel's function of first kind
Parameter β expressed as[2] :

$\beta = \alpha\left[1-\left(\frac{2n}{N-1}\right)^2\right]^{0.5}$ ----(19)

β defines the shape of window and also controls the tradeoff between the amplitude of side lobes and width of main lobe. Larger value of β reduces the window side lobe and therefore ripple also reduce in passband and stopband region.
There are two specific reasons that make Kaiser window better than others :
Window function with minimum stopband attenuation (A_s= $-20\log_{10}\delta_s$) has minimum main lobe width of filter with parameter β as[7]:

$$\beta=\begin{bmatrix} 0.1102(A_s-8.7) & A_s>50 \\ 0.5842(A_s-21)^{0.4}+0.07886(A_s-21) & 21\le A_s\le 50 \\ 0 & A_s\le 50 \end{bmatrix} \quad \text{----(20)}$$

Second is length of filter(N) reduce the main lobe width and achieve desired transition region (Δω) and stopband attenuation (A_s) as[7] :

$N = \frac{(A_s-7.95)}{14.6(\Delta\omega)}$ ---(21)

5.ASSIGN PARAMETERS TO DESIGN FIR LOW PASS FILTER :

In this paper some specified parameters given as :
Passband ripple = 0.03
Stopband ripple = 0.01
Passband frequency = 450 hz
Stopband frequency = 580 hz
Sampling frequency = 1.5 khz
Filter order =177

These parameters used to getting frequency-gain characteristic and frequency-phase characteristic. Let two frequencies f1=450hz,f2=650hz are used as mixed input sinusoidal signal as[3] :

$S(t) = \sin(2*\pi*f_1*t)+\sin(2*\pi*f_2*t)$ ----(22)
$S(t) = \sin(2*\pi*450*t) + \sin(2*\pi*650*t)$.

This sinusoidal input signal used to analyse the time and frequency response in before filtering and after filtering region.

5.1 Simulation Result of Low Pass Filter :

RECTANGULAR WINDOW :

Fig.2 shows that range of pass-band within 0-600Hz and thereafter stop band follows. After filtering process of fig-4 describe that pass-band frequencies i.e. 450Hz and 1050Hz having amplitude level of 23db and frequencies that lies in stopband range are being filtered. In passband region linear phase response is available employing LPF characteristic.

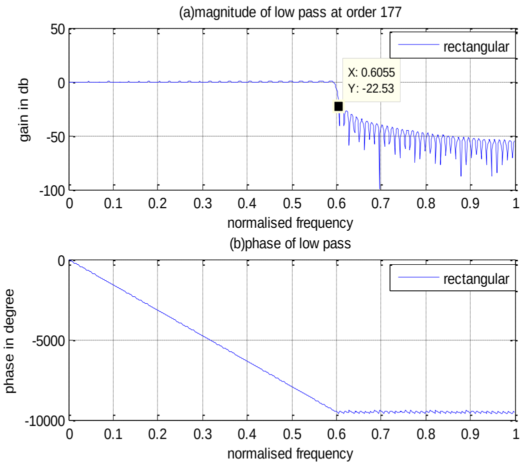

Fig.-2: *Magnitude and phase response of rectangular window*

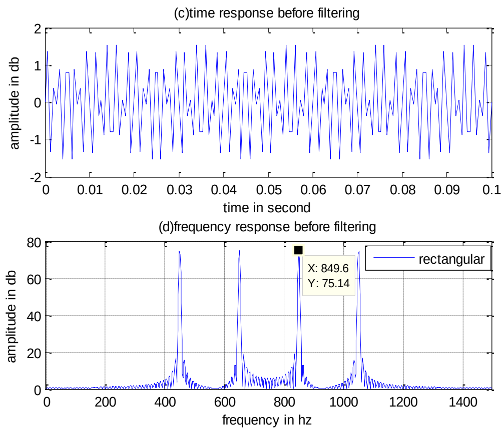

Fig.-3: *Before filtering time and frequency response of rectangular window*

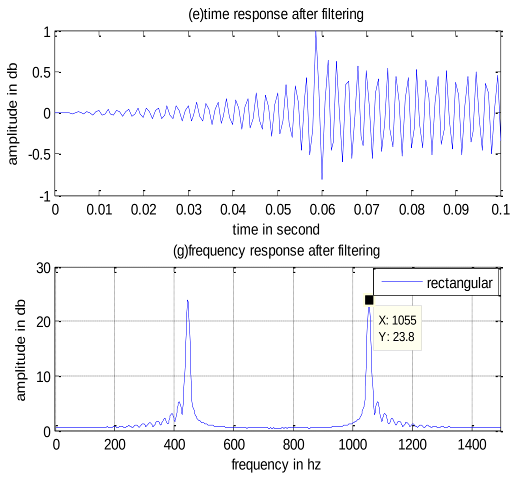

Fig.-4: *After filtering time and frequency response of rectangular window*

HAMMING WINDOW

Fig-5 projects that passband frequency range is 0-600Hz and 610-1000Hz is stopband frequency range. In fig-6 clearly mention that four frequencies 450Hz, 650Hz, 850Hz and 1050Hz with amplitude of 75db are used for filtering process. Phase is linear in passband region, stressing the LPF performance.

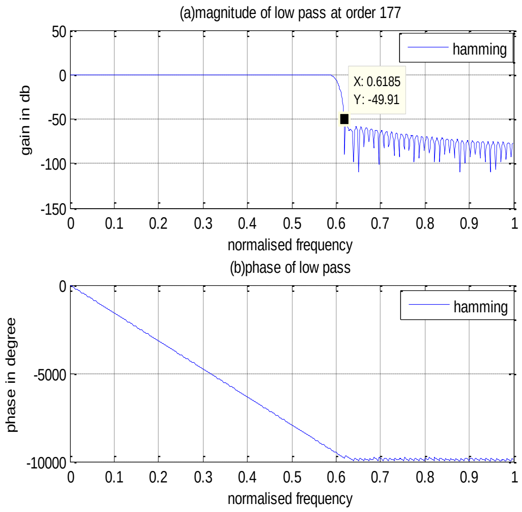

Fig.-5: *Magnitude and phase response of hamming window*

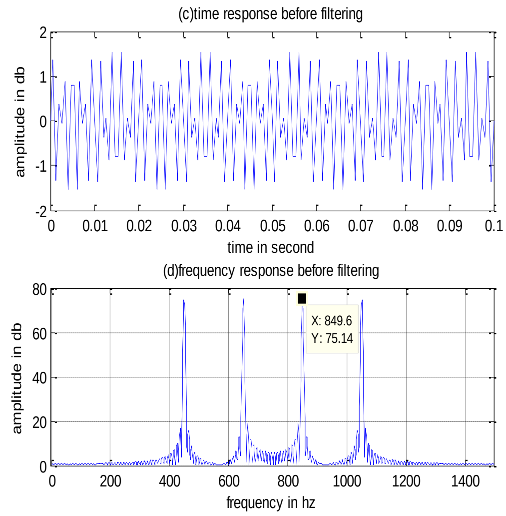

Fig.-6: *Before filtering time and frequency response of hamming window*

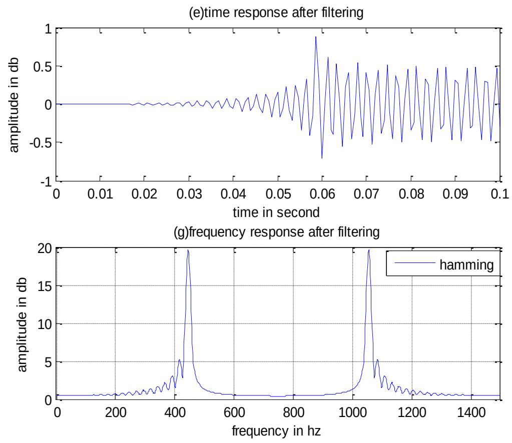

Fig.-7: *After filtering time and frequency response of hamming window*

KAISER WINDOW

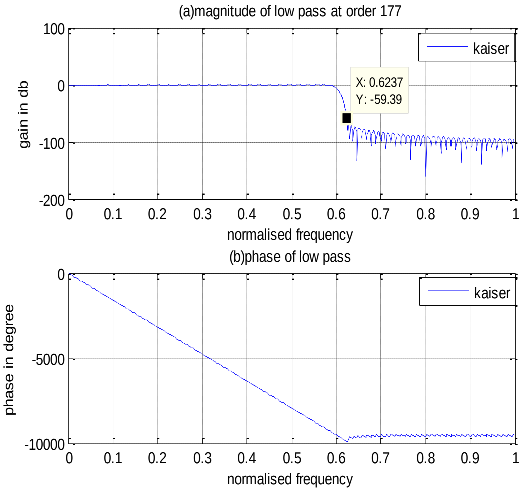

Fig.-8: *Magnitude and phase response of Kaiser Window (β = 6.8)*

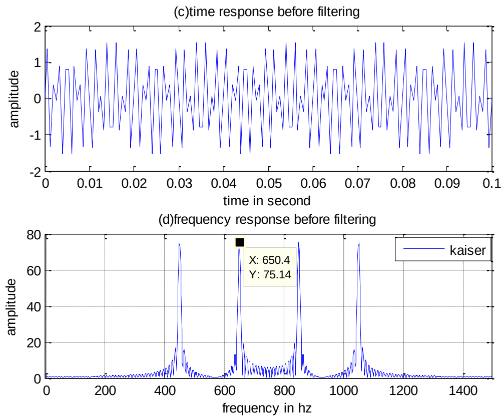

Fig.-9: *Before filtering time and frequency response of Kaiser Window*

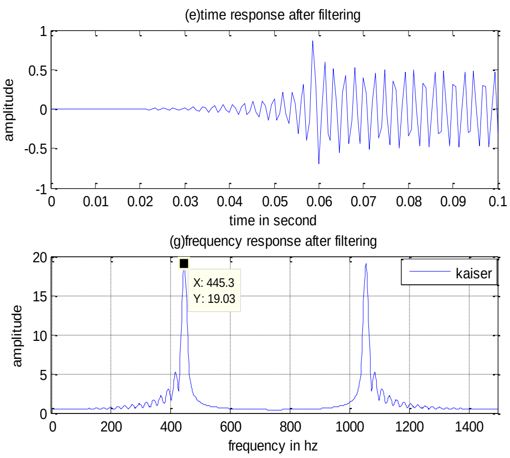

Fig.-10: *After filtering time and frequency response of Kaiser Window*

Magnitude response(Fig.-8) of Kaiser Window shows that passband exist from 0-600Hz and stopband lies thereafter.Similar characteristics are also observed for this window function like earlier counterparts.So passband frequencies kept in after filtering region with amplitude of

19-20 dB.

APPLICATION

Simulation result of low pass FIR filter evidently shows that this filter is used to minimize the noise level and address frequency boosting problem in vast diverse area of signal processing. This design of low pass filtering is primarily used to discard the high frequency spectrum of speech signal and get a desired response in the lower part of spectrum. This paper reviews the results as obtained using the robust MATLAB tool.

CONCLUSION

This paper shows the implementation issues related to a low pass FIR filter encorporating the standard rectangular, hamming and Kaiser windows. Hamming window comparatively provides less ripple and phase is more linear than rectangular and Kaiser Window. So hamming window is considered as stable window compared to other two.. On the other hand, Kaiser window method is preferable for designing of FIR filter because of its design flexibility and optimal characteristics. The authors would like to thank their institutes for providing the facilities to carry out the research review work .

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